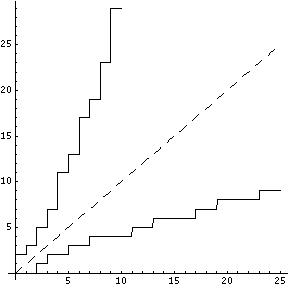
Introduction to Inverse Sequences

In this seminar, we only deal with non-decreasing (but constant only finite intervals) and non-negative integer sequence .

1. Define and point (n,) in x-y plane.
2. Draw horizontal segments connecting the points (n-1,) and (n,) for n>0.
3. Also, draw vertical segments connecting the points (n,) and (n,) for n.
4. Flip symmetrically this drawing with respect to y=x.

Then we get another drawing and a new interesting sequence.



***Definition.*** Let be a non-decreasing and non-negative integer sequence. Then an (geometric) inverse sequence is a sequence such that defined by the step in introduced at the beginning. Let denote that the (geometric) inverse sequence of .

***Remark.*** Let be a non-decreasing and non-negative integer sequence. Then an (geometric) inverse sequence uniquely exists.

***Theorem 1.*** Let be a non-decreasing and non-negative integer sequence. Then is the number of integers less than k in for

***Proof.*** Let and where . Then

Since , there are elements less than in , and implies there is no element such that greater than or equal to (greater than ) and less than in .

There are element less than in .

***Examples.*** (1) **=**1, 2, 3, 4, 5, 6, 7, 8,

**=**0, 1, 2, 3, 4, 5, 6, 7,

(2) **=**2, 3, 5, 7, 11, 13, 17, 19,

**=**0, 0, 1, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6,

(3) **=**1, 1, 2, 3, 5, 8, 13, 21, 34,

**=**0, 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 6, 6, 7,

***Remark.*** We can guess enough about .

***Examples.*** (1)**=**0, 1, 2, 3, 4, 5, 6, 7,

**=**1, 2, 3, 4, 5, 6, 7, 8,

(2)**=**0, 0, 1, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6,

**=**2, 3, 5, 7, 11, 13,

(3) **=**0, 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 6, 6, 7,

=1, 1, 2, 3, 5, 8, 13,

***Theorem 2.*** Let be a non-decreasing and non-negative integer sequence. Then the inverse sequence of is .

***Proof.*** Let satisfy that and arrange the sequence .

Suppose there are elements equal to and elements equal to .

By ***Theorem 1***, we can also arrange .

Note that ,

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and .

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and .

***Reference.***

Tanya Khovanova, How to Create a New Integer Sequence, (2007)

http://www.tanyakhovanova.com/Sequences/CreatingNewSequences.html#inverse